[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1591

Unique Paper Code : 2352011102

Name of the Paper : DSC-2 : Elementary Real Analysis

Name of the Course : **B.Sc. (H) Mathematics (UGCF-2022)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **three** parts from each question.

3. **All** questions carry equal marks.

1. (a) Let $a \geq 0$, $b \geq 0$ prove that $a^2 \leq b^2 \Leftrightarrow a \leq b$.

(b) Determine and sketch the set of pairs (x, y) on $\mathbb{R} \times \mathbb{R}$ satisfying the inequality $|x| \leq |y|$.

(c) Find the supremum and infimum, if they exist, of the following sets :

(i) $\left\{\sin\frac{n\pi}{2} : n \in \mathbb{N}\right\}$

(ii) $\left\{\left(\frac{1}{x} : x > 0\right)\right\}$

(d) Show that Sup $\left\{1+\frac{1}{n} : n \in \mathbb{N}\right\} = 2$.

2. (a) Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a > 0$ and let $aS = \{as : s \in S\}$. Prove that

$$\text{Sup }(aS) = a(\text{ Sup } S)$$

(b) If x and y are positive rational numbers with $x < y$, then show that there exists a rational number r such that $x < r < y$.

(c) Show that $\inf\left\{\frac{1}{n} : n \in \mathbb{N}\right\} = 0$.

(d) Show that every convergent sequence is bounded. Is the converse true? Justify.

3. (a) Using definition of limit, show that

$$\lim_{n\to\infty} \frac{n^2+3n+5}{2n^2+5n+7} = \frac{1}{2}$$

(b) Show that if $c > 0$, $\lim_{n\to\infty} (c)^{1/n} = 1$.

(c) Show that, if $x_n \geq 0$ for all n, and $\langle x_n \rangle$ is convergent then $\langle \sqrt{x_n} \rangle$ is also convergent and

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{\lim_{n\to\infty} x_n}$$

(d) Show that every increasing sequence which is bounded above is convergent.

4. (a) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all n. Prove that $\langle x_n \rangle$ is convergent and find its limit.

(b) Prove that every Cauchy sequence is convergent.

(c) Show that the sequence $\langle x_n \rangle$ defined by

$$x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}, \text{ for all } n \in \mathbb{N}$$

is convergent.

(d) Find the limit superior and limit inferior of the following sequences :

(i) $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$, for all $n \in \mathbb{N}$

(ii) $x_n = \left(1 + \frac{1}{n}\right)^{n+1}$, for all $n \in \mathbb{N}$

5. (a) Show that if a series Σa_n converges, then the sequence $\langle a_n \rangle$ converges to 0.

(b) Determine, if the following series converges, using the definition of convergence, $\sum \log\left(\frac{a_n}{a_{n+1}}\right)$ given that $a_n > 0$ for each n, $\lim_{n\to\infty} a_n = a$, $a > 0$.

(c) Find the rational number which is the sum of the series represented by the repeating decimal $0.\overline{987}$.

(d) Check the convergence of the following series :

(i) $\sum \frac{1}{2^n + n}$

(ii) $\sum \sin\left(\frac{1}{n^2}\right)$

6. (a) State the Root Test (limit form) for positive series. Using this test or otherwise, check the convergence of the following series

(i) $\sum\left(n^{1/n}-1\right)^{n}$

(ii) $\sum\left(\frac{n^{n^2}}{(n+1)^{n^2}}\right)$

(b) Check the convergence of the following series :

(i) $\sum_{n=2}^{\infty}\left(\frac{1}{n\log n}\right)$

(ii) $\sum\left(\frac{n!}{n^n}\right)$

(c) Define absolute convergence of a series. Show that every absolutely convergent series is convergent. Is the converse true? Justify your answer.

(d) Check the following series for absolute or conditional convergence :

(i) $\sum(-1)^{n+1}\left(\frac{n}{n(n+3)}\right)$

(ii) $\sum(-1)^{n+1}\left(\frac{1}{n+1}\right)$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1553

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 : Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) (i) Find a cubic equation with rational coefficients having the roots

$$\frac{1}{2}, \frac{1}{2}+\sqrt{2}, \text{ stating the result used.}$$

(ii) Find an upper limit to the roots of

$x^5 + 4x^4 - 7x^2 - 40x + 1 = 0.$ (4+3.5)

P.T.O.

(b) Find all the integral roots of

$$x^4 + 4x^3 + 8x + 32 = 0. \qquad (7.5)$$

(c) Find all the rational roots of

$$y^4 - \frac{40}{3}y^3 + \frac{130}{3}y^2 - 40y + 9 = 0. \qquad (7.5)$$

2. (a) Express arg $(\overline{z})$ and arg $(-z)$ in terms of arg(z). Find the geometric image for the complex number z, such that $\arg(-z) \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. (2+2+3.5)

(b) Find $|z|$, arg z, Arg z, arg $\overline{z}$, arg(−z) for $z = (1 - i)(6 + 6i)$ (7.5)

(c) Find the cube roots of $z = 1 + i$ and represent them geometrically to show that they lie on a circle of radius $(2)^{1/6}$. (7.5)

3. (a) Solve $y^3 - 15y - 126 = 0$ using Cardan's method. (7.5)

(b) Let n be a natural number. Given n consecutive integers, a, a + 1, a + 2, ..., a + (n−1), show that one of them is divisible by n. (7.5)

(c) Let a and b be two integers such that gcd(a, b) = g. Show that there exists integers m and n such that g = ma + nb. (7.5)

4. (a) Let a be an integer such that a is not divisible by 7. Show that $a \equiv 5^k \pmod 7$ for some integer k. (7.5)

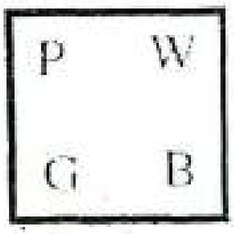
(b) Let a and b be two integers such that 3 divides $(a^2 + b^2)$. Show that 3 divides a and b both. (7.5)

(c) Solve the following pair of congruences, if possible. If no solution exists, explain why? (7.5)

$x + 5y \equiv 3 \pmod 9$

$4x + 5y \equiv 1 \pmod 9$

5. (a) Consider a square with four corners labelled as follows :



Describe the following motions graphically:

(i) R_0 = Rotation of 0 degree.

(ii) R_{90} = Rotation of 90 degrees counterclockwise.

(iii) R_{180} = Rotation of 180 degrees counterclockwise.

(iv) R_{270} = Rotation of 270 degrees counterclockwise.

(v) H = Flip about horizontal axis.

(vi) V = Flip about vertical axis.

(vii) D = Flip about the main diagonal.

(viii) D1 = Flip about the other diagonal.

Identify the motion that can act as identity under the composition of two motions. Further, find out the inverse of each motion. (3.5+1+3)

(b) Show that the set $G = \{f_1, f_2, f_3, f_4\}$, is a group under the composition of functions defined as, $fog(x) = f(g(x))$ for f, g in G, where $f_1(x) = x$, $f_2(x) = -x$, $f_3(x) = 1/x$, $f_4(x) = -1/x$ for all non-zero real number x. (7.5)

(c) Define the inverse of an element in a group G. Show that $(a.b)^{-1} = b^{-1}a^{-1}$ for all a, b in G. Further show that if $(a.b)^{-1} = a^{-1}.b^{-1}$ for all a, b in G, then G is Abelian. (4+3.5)

6. (a) Define Z(G), the center of a group G. Show that Z(G) is a subgroup of G. (2+5.5)

(b) Define order of an element a in group G. Further show that if order of a is n, and $a^m = e$, where m is an integer, then n divides m. (2+5.5)

(c) Find the generators of the cyclic group Z_{30}. Further describe all the subgroups of Z_{30} and find the generators of the subgroup of order 15 in Z_{30}. (2+3.5+2)

[This question paper contains 8 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 1629

Unique Paper Code : 2352011103

Name of the Paper : DSC-3: Probability and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculators and statistical tables is permitted.

1. (a) The following table gives the accompanying specific gravity values for various wood types used in construction. Construct a stem and leaf display and comment on any interesting features of the display

.31	.35	.36	.36	.37	.38	.40	.40	.40
.41	.41	.42	.42	.42	.42	.42	.43	.44
.45	.46	.46	.47	.48	.48	.48	.51	.54
.54	.55	.58	.62	.66	.66	.67	.68	.75

(b) The following data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine. Compute the Median, Upper Fourth (third quartile) and Lower Fourth (first quartile)

1.6	2.0	2.6	3.0	3.9	3.5	4.5	4.6	4.8	5.0

(c) The following table gives the data on oxidation-induction time (measured in minutes) for various commercial oils.

87	103	130	160	180	195	132	145	211	105

(i) Calculate the sample variance and standard deviation.

(ii) If the observations were re-expressed in hours, what would be the resulting values of the sample variance and sample standard deviation? Answer without reperforming the calculations.

2. (a) If A and B are any two events, then show that $P(A \cap B') = P(A) - P(A \cap B)$. Hence or otherwise prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will not be discovered?

(c) State Baye's Theorem A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two- night visit. In addition, 10% of day visitors ultimately make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit?

3. (a) In a group of five potential blood donors a, b, c, d, and e, only a and b have Opositive (O+) blood type. Five blood samples, one from each individual, will be typed in random order until an O+ individual is identified. Let the random variable Y = the number of typings necessary to identify an O+ individual.

(i) Find the probability mass function (pmf) of Y.

(ii) Draw the line graph and probability histogram of the pmf.

(b) The n candidates for a job have been ranked 1,2,3, ... n. Each candidate has an equal chance of being selected for the job. Let the random variable X be defined as

X = the rank of a randomly selected candidate

(i) Find the probability mass function (pmf) of X.

(ii) Compute E(X) and V(X).

(c) For any random variable X, prove that $V(aX + b) = a^2V(X)$ and $\sigma_{aX+b} = |a|\sigma_X$.

4. (a) The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with probability density function (pdf)

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

(i) the cumulative density function (cdf) of sales

(ii) E(X)

(iii) V(X)

(iv) σ_X

(b) The reaction time for an in-traffic response to a brake signal from standard brake lights can be modelled with a normal distribution having mean value 1.25 sec and standard deviation of 0.46 sec.

What is the probability that the reaction time is between 1.00 sec and 1.75 sec? If 2 sec is a critical long reaction time, what is the probability that actual reaction time will exceed this value?

(c) If X is a binomially distributed random variable with parameters n and p, prove that

(i) $E[X] = np$

(ii) $V[X] = np(1 - p)$

5. (a) If 75% of all purchases in a certain store are made with a credit card and the random variable, X = number among ten randomly selected purchases made with a credit card is a Binomial variate, then determine

(i) $E(X)$

(ii) $V(X)$

(iii) σ_X

(iv) The probability that X is within 1 standard deviation of its mean value.

(b) Let X denote the amount of time a book on two-hour reserve is actually checked out, and suppose the cumulative density function is

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

(i) Calculate $P(.5 \le X \le 1)$.

(ii) What is the median checkout duration $\tilde{\mu}$?

(iii) Obtain the density function $f(x)$.

(c) Theamount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size c will ensure that overflow occurs only 0.5% of the time?

6. (a) Toughness and fibrousness of asparagus are major determinants of quality. This was reported in a study with the following data on x = shear force (kg) and y = percent fiber dry weight.

X	46	48	55	57	60	72	81	85	94	109
y	2.18	2.10	2.13	2.28	2.34	2.53	2.28	2.62	2.63	2.50

(i) Calculate the value of the sample correlation coefficient. Based on this value, how would you describe the nature of relationship between the two variables?

(ii) If shear force is expressed in pounds, what happens to the value of r? Why?

(b) An experiment was performed to investigate how the behavior of mozzarella cheese varied with temperature. The following observations on x = temperature and y = elongation(%) at failure of the cheese.

X	59	63	68	72	74	78	83
y	118	182	247	208	197	135	132

(i) Determine the equation of the estimated regression line using the principle of least square.

(ii) Estimate the elongation at failure of the cheese when the temperature is 70.

(c) The inside diameter of a randomly selected piston ring is a random variable with mean value of 12 cm and standard deviation 0.04 cm. If $\overline{X}$ is the sample mean diameter for a random sample of n = 16 rings,

(i) where is the sampling distribution of $\overline{X}$ centered,

(ii) what is the standard deviation of the $\overline{X}$ distribution.

(iii) How likely is it that the sample mean diameter exceeds 12.01?

[This question paper contains 8 printed pages]

Your Roll No. ....

Sr. No. of Question Paper : 831

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.Sc. (Physical Science and Mathematical Science) with Operational Research and Bachelor of Arts**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting **two** parts form each question.

3. **All** questions carry equal marks.

P.T.O.

1. (a) Show that the homogeneous equation

$$(Ax^2 + Bxy + Cy^2)dx + (Dx^2 + Exy + Fy^2) = 0$$

is exact if and only if B = 2D and E = 2C. Also solve the initial value problem

$$(2x^2y^2 - y^3 + 2x)dx + (2x^3y - 3xy^2 + 1)dy = 0,$$
$y(-2) = 1.$ (7½)

(b) Show that $y = 4e^{2x} + 2e^{-3x}$ is a solution of the initial value problem

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0 \quad y(0) = 6, \quad y'(0) = 2.$$

Is $y = 2e^{2x} + 4e^{-3x}$ also a solution of this problem? Explain why or why not. (7½)

(c) Consider the equation $a\left(\frac{dy}{dx}\right) + by = ke^{-\lambda x}$, where a, b and k are positive constants and λ is nonnegative constant.

(i) Solve this equation.

(ii) Show that if $\lambda = 0$ every solution approaches $\frac{k}{b}$ as $x \to \infty$, but if $\lambda > 0$ every solution approaches 0 as $x \to \infty$.

(7½)

2. (a) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = 100$ and family of Parabolas $y = 10x^2$. (7½)

(b) The population x of a certain city satisfies the logistic law

$$\frac{dx}{dt} = \frac{1}{100x} - \frac{1}{(10)^8}x^2$$

where time t is measured in years. Given that the population of this city is 100,000 in 1980, determine the population as a function of time for t > 1980. In particular answer the following questions :

(i) What will be the population in 2000?

(ii) In what year the does the 1980 population double? (7½)

(c) Solve the Homogeneous differential equation $2r(s^2+1)dr + (r^4+1)ds = 0.$ (7½)

3. (a) Find the general solution of

$$\left(x^2+2x\right)\frac{d^2y}{dx^2}-\frac{2(x+1)dy}{dx}+2y=1$$

Given that $y = (x + 1)$ and $y = (x + 1)^2$ are linearly independent solutions of the corresponding homogeneous equation. (7½)

(b) Find the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 2x \ln x, \quad x > 0 \quad (7½)$$

(c) Define Quasi–linear, Semi–linear and linear first order partial differential equation and give one example each. Also show that a family of spheres $x^2 + y^2 + (z - c)^2 = r^2$ satisfies the first order linear partial differential equation $y\frac{dz}{dx} - x\frac{dz}{dy} = 0$.

(7½)

4. (a) Use the method of variation of parameter to find a particular solution of the differential equation :

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}. \quad (7½)$$

P.T.O.

(b) Use the method of undetermined coefficients to find the particular solution of the differential equation :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 16x - 12e^{2x}. \qquad (7\tfrac{1}{2})$$

(c) Given that $y = e^{2x}$ is a solution of

$$(2x+1)\frac{d^2y}{dx^2} + 4(x+1)\frac{dy}{dx} + 4y = 0.$$

Find a linearly independent solution by reducing the order. Write the general solution. (7½)

5. (a) Find the solution of Cauchy problem for first order PDE.

$$2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} + z = 0 \text{ with } u(x, 0) = \sin x \qquad (7\tfrac{1}{2})$$

(b) Find the Solution of characteristic equation for the first order PDE.

$$\frac{\partial u}{\partial x} + 2x\frac{\partial u}{\partial y} = 2xu \text{ with } u(x, 0) = x^2$$ (7½)

(c) Find the general solution of the equation :

$$(cy - bz)zx + (az - cz)zy = bx - ay$$ (7½)

6. (a) Solution general solution of Cauchy problem for first order PDE.

$$\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = \left(\frac{y-1}{2x^2}\right)^2 \text{ with } u(0, y) = \exp(y)$$ (7½)

(b) Find the general solution of the partial differential equation :

$$x^2(y - u)u_x + y^2(u - x)u_y = u^2(x - y)$$ (7½)

(c) Reduce the equation :

$y^2 u_{xx} + x^2 u_{yy} = 0$, $x \neq 0$, $y \neq 0$, find the general solution. (7½)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1534

Unique Paper Code : 2352012301

Name of the Paper : Group Theory

Name of the Course : **B.Sc. (H) Mathematics – DSC**

Semester : III

Duration : 3 Hours   Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting **two** parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

5. Use of Calculator is not allowed.

1. (a) Derive a formula for finding the order of a permutation of a finite set written in disjoint cycle form. Let $\beta = (1,3,5,7,9)\ (2,4,6)\ (8,10)$. What is the smallest positive integer for which $\beta^m = \beta^{-5}$?

(b) Let $\alpha, \beta \in S_n$. Prove that $\alpha\beta$ is even if and only if either both α and β are even or both α and β are odd.

(c) Suppose H is a subgroup of S_n of odd order. Prove that H is a subgroup of A_n.

2. (a) Let H and K be normal subgroups of a group G such that $H \cap K = \{e\}$, then prove that the elements of H and K commute. Give an example of a non-Abelian group whose all subgroups are normal.

(b) Let G be group and suppose that N is a normal subgroup of G and H is any subgroup of G. Prove that $N \cap H$ is a normal subgroup of G. Justify this statement with an example too.

(c) Suppose H and K are subgroups of a group G. If aH is a subset of bH, then prove that H is a subset of K. Is the converse true except when a and b are identity? Justify your answer with an example.

3. (a) State and prove first isomorphism theorem. Show that $\mathbb{Z}/n\mathbb{Z}$ is isomorphic to $\mathbb{Z}_n$, for all $n \in \mathbb{N}$.

(b) Let ϕ be an isomorphism from a group G onto a group G′, prove that

(i) G is cyclic if and only if G′ is cyclic.

(ii) for all elements $g \in G$, the order of g is equal to order of $\phi(g)$.

(c) (i) Let G be a group, prove that the mapping ϕ defined as

$$\phi(g) = g^{-1} \text{ for all } g \in G$$

is an automorphism if and only if G is an Abelian group.

(ii) Suppose ϕ is a homomorphism from a group G onto a group G′, prove that

$$\phi(Z(G)) \subseteq Z(G').$$

Here, Z(G) denotes the center of G.

4. (a) (i) Suppose that G is a finite Abelian group and G has no element of order 2. Show that the mapping ϕ defined as

$$\phi(g) = g^2 \text{ for all } g \in G$$

is an automorphism. Is this mapping ϕ an automorphism when G is an infinite group and has no element of order 2? Justify your answer.

(ii) Prove that a homomorphism ϕ from a group G onto a group G′ is one-one if and only if $\ker \phi = \{e\}$, where e is the identity element of G.

(b) (i) Suppose ϕ is a homomorphism from a group G to a group G′ and $g \in G$ such that $\phi(g) = g'$, prove that

$$\phi^{-1}(g') = g \ker \phi.$$

(ii) For $\mathbb{C}^*$, the multiplicative group of non-zero complex numbers, prove that the mapping ϕ defined as

$$\phi(z) = z^6 \text{ for all } z \in \mathbb{C}^*$$

is a homomorphism. Also, find the kernel of ϕ.

(c) (i) Prove that every normal subgroup of a group G is the kernel of a homomorphism of G.

(ii) Suppose that ϕ is a homomorphism from U(40) to U(40) and its kernel is given by $\ker \phi = \{1, 9, 17, 33\}$. If $\phi(11) = 11$, find all elements of U(40) that are mapped to 11.

5. (a) Find all subgroups of order 3 in $\mathbb{Z}_4 \oplus \mathbb{Z}_3$.

(b) Let G and H be finite cyclic groups. Then $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime

(c) Using the concept of external direct product, determine the last two digits of the number 23^{123}.

6. (a) Determine the number of elements of order 15 and the cyclic subgroups of order 15 in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{26}$.

(b) Define the internal direct product of n normal subgroups of a group. If a group G is the internal direct product of a finite number of normal subgroups

$H_1, H_2, \ldots, H_n$, then show that G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$.

18 JUN 2024

Sr. No. of Question Paper : 1572

Unique Paper Code : 2352012302

Name of the Paper : DSC-8 : Riemann Integration

Programme : B.Sc. (Hons.) Mathematics (NEP-UGCF 2022)

Semester : III

Duration : 3 Hours   Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory. Attempt any **Three** parts from each question.
3. All questions carry equal marks.

1. (a) Let $f: [-1,1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 2, & if\ x \in \mathbb{Q} \\ 3, & if\ x \notin \mathbb{Q} \end{cases}$$

Show that f is not integrable on [-1,1].

(b) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if f is integrable on $[a,b]$, then for each $\varepsilon > 0$, there exists a $\delta > 0$ such that $U(f,P) - L(f,P) < \varepsilon$ for every partition P of $[a,b]$ with mesh$(P) < \delta$.

(c) Let $f(x) = 3x + 2$ over the interval [1,3]. Let P be a partition of [1,3] given by $P = \{1, 3/2, 2, 3\}$. Compute $L(f,P), U(f,P)$ and $U(f,P) - L(f,P)$.

(d) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if P and Q are any partitions of $[a,b]$, then $L(f,P) \leq U(f,Q)$. Hence show that $L(f) \leq U(f)$.

(e)

2. (a) Prove that a bounded function f is integrable on $[a,b]$ if and only if there exists a sequence of partitions $(P_n)_{n\in\mathbb{N}}$ of $[a,b]$, satisfying $\lim[U(f,P_n) - L(f,P_n)] = 0$.

(b) Suppose that a function f defined on $[a,b]$ is integrable on $[a,c]$ and $[c,b]$, where $c \in (a,b)$. Prove that f is integrable on $[a,b]$ and that $\int_a^b f = \int_a^c f + \int_c^b f$.

(c) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if f is Riemann integrable on $[a,b]$, then it is (Darboux) integrable on $[a,b]$, and that the values of the integrals agree.

(d) For $t \in [0,1]$, let $F(t) = \begin{cases} 0 & for\ t < 1/3 \\ 1 & for\ t \geq 1/3. \end{cases}$

Let $f(x) = x^2$, where $x \in [0,1]$. Show that f is F-integrable and that $\int_0^1 f\, dF = f(1/3)$.

3. (a) Prove that every continuous function on $[a,b]$ is integrable on $[a,b]$.

(b) State and prove the Intermediate Value Theorem for Integrals.

(c) (i) Show that $\left|\int_{-2\pi}^{2\pi} x^2 \cos^8(e^x)\, dx\right| \le \frac{16\pi^3}{3}$.

(ii) Give an example of a function f on [0, 1] that is not integrable for which $|f|$ is integrable on [0, 1].

(d) Suppose that f and g are continuous functions on $[a, b]$ such that $\int_a^b f\, dx = \int_a^b g\, dx$. Prove that there exists x in $[a, b]$ such that $f(x) = g(x)$.

4. (a) If f and g are two integrable functions on $[a, b]$, then prove that $(f + g)$ is also integrable on $[a, b]$.

(b) Prove that every piecewise monotonically increasing function on $[a, b]$ is integrable on $[a, b]$.

(c) State Fundamental Theorem of Calculus-II. Hence or otherwise evaluate $\lim_{h \to 0} \frac{1}{h} \int_4^{4+h} e^{t^2}\, dt$.

(d) Let f be defined on $\mathbb{R}$ as

$$f(t) = \begin{cases} t & \text{for } t < 0 \\ t^2 + 1 & \text{for } 0 \le t \le 2 \\ 0 & \text{for } t > 2. \end{cases}$$

Determine the function $F(x) = \int_0^x f(t)\, dt$.

(i) At what points F is continuous?

(ii) At what points F is differentiable? Calculate F' at the points of differentiability.

5. (a) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval [0, 2] is rotated about the line $y = -1$.

(b) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 4$, $x = 9$ and the x-axis is revolved about the y-axis.

(c) Find the exact arc length of the curve $y = x^{\frac{2}{3}}$ from x = 1 to x = 8.

(d) The circle $x^2 + y^2 = r^2$ is rotated about the x-axis to obtain a sphere. Find the surface area of the sphere.

6. (a) Discuss the convergence of following improper integrals:

(i) $\int_0^1 \frac{1}{x \ln x}\, dx$ (ii) $\int_1^\infty \frac{dx}{\sqrt{x^3 + x}}$.

(b) Find a function f such that $\int_1^\infty f$ converges, but $\int_1^\infty \sqrt{f}$ does not converge. Justify your answer.

(c) Show that the improper integral $\int_0^1 t^{p-1}(1-t)^{q-1}\, dt$ converges if and only if p and q are positive.

(d) Show that

(i) $\Gamma(p+1) = p\,\Gamma(P)$ for all $p > 0$,

(ii) $\Gamma(n) = (n-1)!$ for all $n \in \mathbf{N}$.

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4404

Unique Paper Code : 32351302

Name of the Paper : Group Theory – I

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question from **Q2** to **Q6**.

4. In the question paper, given notations have their usual meaning unless until stated otherwise.

P.T.O.

1. Give **short** answers to the following questions. Attempt any **six**.

(i) Find an element X in D_4 such that $R_{90}VXH = D'$.

Where R_{90} = Rotation of 90°, V = Flip about a vertical axis, H = Flip about a horizontal axis, D′ = Flip about the other diagonal.

(ii) Is G = {1,2,3,4,5} a group under multiplication modulo 6? In general when is G = {1,2,...,n-1}; $n \geq 2$, a group under multiplication modulo n? Answer both in a few lines.

(iii) Can a non-Abelian group have a non-trivial Abelian subgroup? Give short answer in few lines.

(iv) Let G be a group such that $x = x^{-1}$, for all $x \in G$. Prove that G is Abelian.

(v) Give an example of a non-cyclic group, whose every proper subgroup is cyclic.

(vi) Prove that a group of order 4 is Abelian.

(vii) List all the generators of (Z, +), Z_7 and Z_8.

(viii) For any integer n > 2, show that there are at least two elements in U(n) that satisfy $x^2 = 1$.

(6×2=12)

2. (a) Prove that

$$G = \left\{ \begin{bmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{bmatrix} : a \in \mathbb{R} \right\}$$

is an infinite Abelian group under matrix multiplication.

(b) Define a cyclic subgroup of a group. Is it Abelian or Non-Abelian? Justify your answer. Prove that

$$H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} : n \in Z \right\}$$

is a cyclic subgroup of $GL(2, \mathbb{R})$.

(c) Prove that the subgroup of a cyclic group is cyclic. Find the smallest subgroup of (Z, +) containing 8 and 14. (2×6.5=13)

3. (a) Prove that the order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. (6)

(b) (i) Show that S_7 has an element of order 12. Find one such element.

(ii) Give two reasons why the set of odd permutations in S_n is not a subgroup.

(3+3=6)

(c) (i) Let G = U (24), H = {1,7}. Write all the distinct left cosets of H in G.

(ii) Prove that a group of order 98 can have at the most one subgroup of order 49.

(3+3=6)

4. (a) (i) Let H be a subgroup of G and a and b belongs to G. Then prove that

$aH = bH$ iff $a^{-1}b \in H$

(ii) State Lagrange's Theorem for finite groups and prove that every group of prime order is cyclic. (3+3.5=6.5)

P.T.O.

(b) (i) Let $H = \{1, (12)(34)\}$, $G = A_4$. Show that H is not a normal subgroup of G.

(ii) Is the order of a factor group of an infinite group is infinite? Give example or counter example to support your answer.

(3+3.5=6.5)

(c) (i) Prove that Z (G), the centre of a group G, is always a normal subgroup of G.

(ii) Let G = Z, the group of integers under addition. Write all the elements of factor group Z/20Z of Z. Is this factor group cyclic? Give explanation in support of your answer.

(3+3.5=6.5)

5. (a) If H is a subgroup of a group G and K is a normal subgroup of G, then prove that $H/(H \cap K)$ is isomorphic to HK/K.

(b) Determine the possible homomorphisms from Z_{20} to Z_{10}. Also, find which of the homomorphisms are onto.

(c) Prove or disprove the following by justifying them :

(i) $U(8) \approx Q_8$, the group of Quaternions.

(ii) $U(20) \approx D_4$

(iii) $(Q, +) \approx (Z, +)$ (2×6=12)

6. (a) If ϕ is an isomorphism from a group G onto a group $\overline{G}$, then prove that

$|\phi(g)| = |g|$ for all $g \in G$.

(b) Let $\mathbb{C}$ be the set of complex numbers and

$M = \left\{\begin{bmatrix} a & -b \\ b & a \end{bmatrix} : \quad a, b \in \mathbb{R}\right\}$. Prove that $\mathbb{C}$ and M are isomorphic under addition and that $\mathbb{C}^*$ and M^*, the non-zero elements of M, are isomorphic under multiplication.

(c) Suppose that ϕ is a homorphism from U/(30) to U(30) and that Kerϕ = {1,11}. If $\phi(7) = 7$, find all the elements of U(30) that are mapped to 7. State and prove the result used.

(2×6.5=13)

[This question paper contains 12 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1694

Unique Paper Code : 2353012001

Name of the Paper : Graph Theory

Name of the Course : **B. Sc. (Hons.) Mathematics-DSE**

Semester : III

Duration : 3 Hours   Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All questions has three parts **(a)**, **(b)** and **(c)**. You have to attempt any **two** parts of each question.

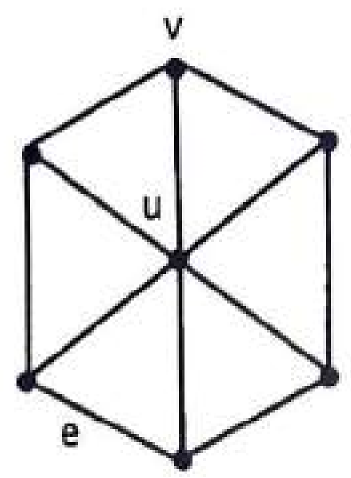
3. **All** questions carry equal marks.

4. Parts of each question to be attempted together.

5. Use of Calculator not allowed.

P.T.O.

1. (a) (i) Define sub-graph of a graph. Draw pictures of the sub-graphs of G\{e}, G\{v} and G\{u} of the following graph G.



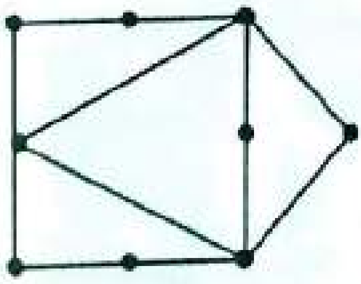
(ii) Determine that if there exist a graph whose degree sequence is 5,4,4,3,2,1. Either draw a graph or explain why no such graph exists.

Draw a graph with degree sequence 4,3,2,2,1.

(4.5,3)

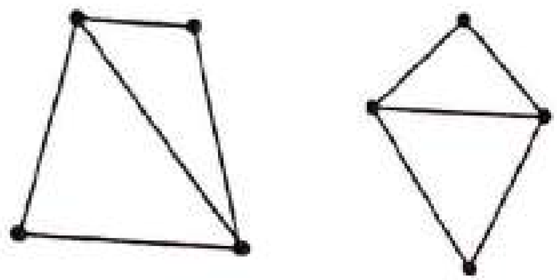
(b) (i) Define a complete graph. Does there exists a graph G with 30 edges and 10 vertices; each of degree 4 or 5. Justify your answer?

(ii) What is a bipartite graph? Determine whether the graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite.



If bipartite then determine whether it is complete bipartite. (3,4.5)

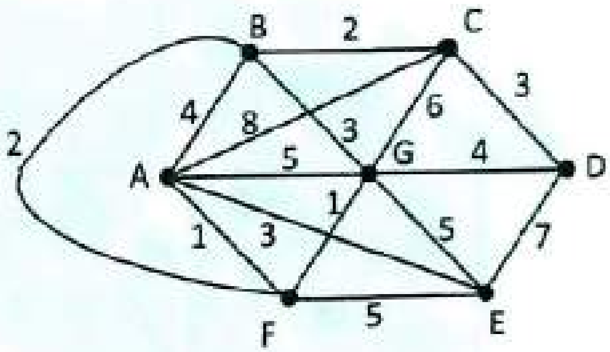
(c) (i) Define the term Isomorphic Graphs. For the below pair of graphs, either label the graphs so as to exhibit an isomorphism or explain why graphs are not isomorphic :



(ii) Solve the Chinese Postman Problem for the graph below :

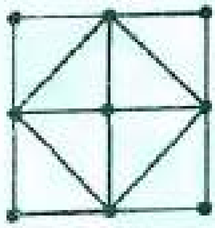
(4,3.5)

2. (a) Apply the improved version of Dijkstra's algorithm to find the length of a shortest path from A to D in the graph shown below. Also find the corresponding shortest path. Label all vertices and write steps.



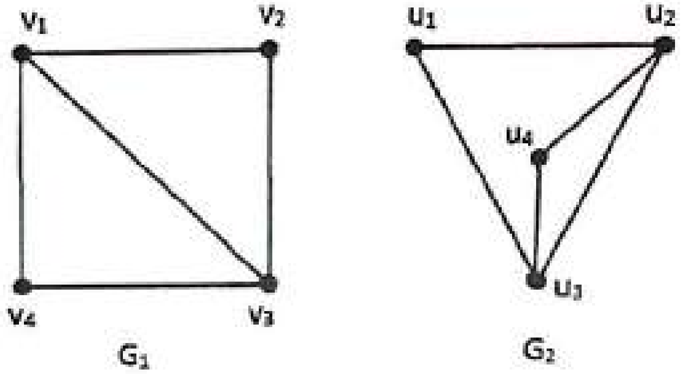
(7.5)

(b) Define Eulerian graph and Hamiltonian graph. Consider the graph G given below. Is it Eulerian? Is it Hamiltonian? Explain your answers.



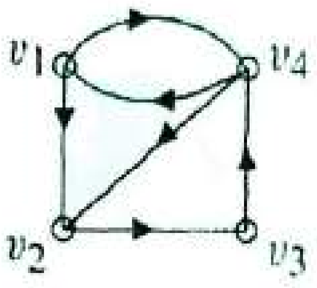
(7.5)

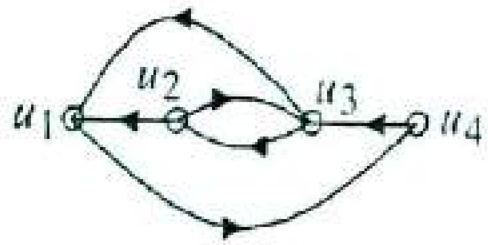
(c) Define adjacency matrix of a graph. Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 given below. Find a permutation matrix P such that $A_2 = PA_1P^T$, thus proving that G_1 and G_2 are isomorphic.



(7.5)

3. (a) Write the definition of a digraph along with an example with 5 vertices. Explain whether the following digraphs are isomorphic or not :





(7.5)

(b) Write the definition of a transitive tournament along with an example. Show that if T is a tournament having a unique Hamiltonian path, then T is transitive. (7.5)

(c) The construction of a certain part in an automobile engine involves four activities : pouring the mold, calibration, polishing, and inspection. The mold is poured first; calibration must occur before the inspection. Pouring the mold takes eight units of time, calibration takes three units, polishing takes six units of time for an uncalibrated product and eight units of time for a calibrated one, and inspection takes two units of time for a polished product and three units of time for an unpolished one.

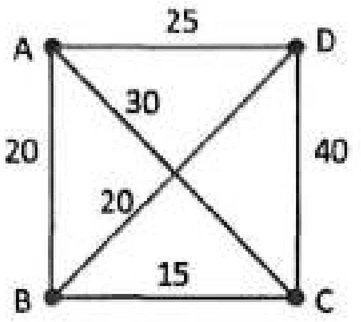
(i) Draw the appropriate directed network that displays the completion of this job.

(ii) What is the shortest time required for this job? Describe the critical path.

(3,4.5)

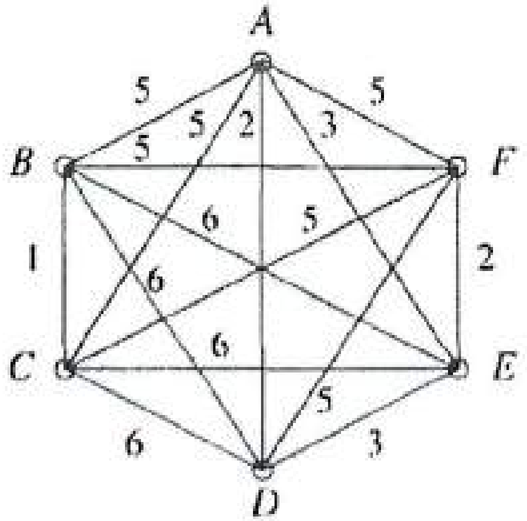
4. (a) (i) Draw all non-isomorphic trees containing 6 vertices.

(ii) Solve the Travelling Salesman's problem for the following graph by making tree that displays all the Hamiltonian Cycles (Start with A) :



(4,3.5)

(b) What is the spanning tree and minimum spanning tree for a connected graph G? Use Kruskal's algorithm to find a spanning tree of minimum total weight for the following graph



What is the weight of the minimum tree and show your steps. (7.5)

(c) (i) Write the definition of a bridge for a graph G along with an example.

(ii) Let G be a connected graph of order n. If every edge of the graph is a bridge, then what is the total number of edges in the graph G. Give an example.

(iii) Give an example of a connected graph G with the properties that every bridge of G is adjacent to an edge that is not a bridge and every edge of G that is not a bridge is adjacent to a bridge. (2.5,2.5,2.5)

5. (a) Define vertex-connectivity and edge-connectivity of a graph. What is the relationship among vertex-connectivity, edge-connectivity and minimum degree of a graph? Verify it for k_4 and $k_{3,3}$.

(7.5)

(b) Define planar and nonplanar graph with example. If G is a connected planar graph with e edges and n vertices, where $n \geq 3$, then prove that $e \leq 3n - 6$. Hence prove that k_5 is nonplanar. (7.5)

(c) State and explain Kuratowski's theorem. Let G be a connected plane graph with e edges and n vertices such that every region of G has at least five edges on its boundary, then show that $3e \leq 5n - 10$. (7.5)

6. (a) State and explain four color problem. Let $\Delta(G)$ be the maximum of the degrees of the vertices of a graph G, then show that $\chi(G) \leq 1 + \Delta(G)$. (7.5)

(b) State and explain Hall's marriage theorem. Let G be a bipartite graph with bipartition sets V_1, V_2 in which every vertex has the same degree k. Show that G has a matching which saturates V_1. (7.5)

(c) Define independent set and edge cover of a graph with example. Compute the maximum size of independent set and minimum size of edge cover in C_5 and k_5 (where C_n is a n-cycle).

(7.5)

[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 1696

Unique Paper Code : 2353012003

Name of the Paper : Number Theory

Name of the Course : **B.Sc. (H) Mathematics – DSE**

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Both parts of a question to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

1. (a) When Mr. Smith cashed a check at his bank, the teller mistook the number of cents for the number of dollars and vice versa. Unaware of this, Mr. Smith spent 68 cents and then noticed to his

surprise that he had twice the amount of the original check. Determine the smallest value for which the check could have been written.

(7.5)

(b) Solve the linear congruence $17x \equiv 3 \pmod{2 \cdot 3 \cdot 5 \cdot 7}$. (7.5)

(c) Verify that $0, 1, 2, 2^2, 2^3, \ldots, 2^9$ form a complete set of residues modulo 11, but that $0, 1^2, 2^2, 3^2, \ldots, 10^2$ do not. (7.5)

2. (a) State and prove Wilson's Theorem. (7.5)

(b) (i) If $\gcd(a, 30) = 1$, show that 60 divides $a^4 + 59$. (4.5)

(ii) Use Fermat's theorem to verify that 17 divides $11^{104} + 1$. (3)

(c) If $n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$ is the prime factorization of $n > 1$, then derive the formulae for $\tau(n)$ and $\sigma(n)$, where $\tau(n)$ denotes the number of positive divisors of n and $\sigma(n)$ denotes the sum of these divisors. Also, show that the product of positive divisors of n is $n^{\frac{\tau(n)}{2}}$. (7.5)

3. (a) State and prove Euler's theorem. Find the last two digits of 3^{256}. (4+3.5)

(b) For each positive integer n, show that $\mu(n)\,\mu(n+1)\,\mu(n+2)\,\mu(n+3) = 0$ Also, show that for any integer $n \geq 3$, $\sum_{k=1}^{n} \mu(k!) = 1$. (4+3.5)

(c) (i) Find the highest power of 9 dividing 901!. (4.5)

(ii) Determine all primitive roots of 3^2. (3)

4. (a) (i) Show that if the integer n has r distinct odd prime factors, then 2^r divides $\varnothing(n)$, where $\varnothing$ is the Euler Phi function. (3.5)

(ii) Use Euler's theorem to show that for any integer a,

$$a^{37} \equiv a \pmod{1729}.$$ (4)

(b) Prove that for any integer *a*, *a* and a^{4n+1} have same last digit. (7.5)

(c) (i) Show that if $F_n = 2^{2n} + 1$, $n > 1$, is a prime, then 2 is not a primitive root of F_n. (3.5)

(ii) Let r be a primitive root of the odd prime p. Show that if $p \equiv 1 \pmod 4$, then $-r$ is also a primitive root of the odd prime p. (4)

5. (a) Define a quadratic residue of an odd prime p.

Let p be an odd prime and $\gcd(a, p) = 1$. Then

P.T.O.

prove a is a quadratic residue of p if and only if

$a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.

Hence show that 3 is a quadratic residue of 23, but a quadratic nonresidue of 31. (1+4+2.5)

(b) (i) Let p be an odd prime and let a and b be integers relatively prime to p. Then prove $\left(\frac{ab}{p}\right) = \left(\frac{a}{p}\right)\left(\frac{b}{p}\right)$. Hence calculate $\left(\frac{1234}{4567}\right)$. (2+3)

(ii) Is $x^2 \equiv -46 \pmod{17}$ solvable ? (2.5)

(c) (i) Show that there are infinitely many primes of the form 6k + 1. (3)

(ii) Solve the quadratic congruence $x^2 \equiv 7 \pmod{3^2}$. (4.5)

6. (a) Encrypt the message HAPPIEST MINDS using the linear cipher

$$C \equiv 5P + 11 \pmod{26}.$$ (7.5)

(b) Use the Hill Cipher

$$C_1 \equiv 4P_1 + 2P_2 \pmod{26}$$
$$C_2 \equiv 3P_1 + 8P_2 \pmod{26}$$

to encipher the message ACT NOW. (7.5)

(c) Encrypt the plaintext message PAINTS using RSA algorithm, with key (2701,7). (7.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1610 **G**

Unique Paper Code : 2352012303

Name of the Paper : Discrete Mathematics

Name of the Course : **B.Sc. (H) – DSC**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

P.T.O.

1. (a) (i) Define covering relation in an ordered set and finite ordered set. Prove that if X is any set, then the ordered set $\wp(X)$ equipped with the set inclusion relation given by $A \leq B$ iff $A \subseteq B$ for all $A, B \in \wp(X)$, a subset B of X covers a subset A of X iff $B = A \cup \{b\}$ for some $b \in X - A$.

(ii) State Zorn's Lemma.

(b) (i) Give an example of an ordered set (with diagram) with more than one maximal element but no greatest element. Specify maximal elements also.

(ii) Define when two sets have the same cardinality. Show that

- $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$
- $\mathbb{Z}$ and $2\mathbb{Z}$

have the same cardinality.

(c) Let $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n. Draw Hasse diagram for the subset $S = \{1,2,4,5,6,12,20,30,60\}$ of $(\mathbb{N}_0, \leq)$. Find elements $a, b, c, d \in S$ such that $a \vee b$ and $c \wedge d$ does not exist in S.

2. (a) Define an order preserving map. In which of the following cases is the map $\varphi : P \to Q$ order preserving?

(i) $P = Q = (\mathbb{N}_0, \leq)$ and $\varphi(x) = nx$ ($n \in \mathbb{N}_0$ is fixed).

(ii) $P = Q = (\wp(\mathbb{N}), \subseteq)$ and φ defined by

$$\varphi(U) = \begin{cases} \{1\}, & 1 \in U \\ \{2\}, & 2 \in U \text{ and } 1 \notin U, \\ \emptyset, & \text{otherwise} \end{cases}$$

P.T.O.

where $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ iff m divides n and $\wp(\mathbb{N})$ be the power set of $\mathbb{N}$ equipped with the partial order given by $A \leq B$ iff $A \subseteq B$ for all $A, B \in \wp(\mathbb{N})$.

(b) For disjoint ordered sets P and Q define order relation on $P \cup Q$. Draw the diagram of ordered sets (i) 2×2 (ii) $3 \cup \overline{3}$ (iii) $M_2 \oplus M_3$ where $M_n = 1 \oplus \bar{n} \oplus 1$.

(c) Let $X = \{1,2,\ldots, n\}$ and define $\varphi: \wp(X) \to 2^n$ by $\varphi(A) = (\varepsilon_1,\ldots, \varepsilon_n)$ where

$$\varepsilon_i = \begin{cases} 1 & \text{if } i \in A \\ 0 & \text{if } i \notin A \end{cases}$$

Show that φ is an order-isomorphism.

3. (a) Let L and K be lattices and $f: L \to K$ a lattice homomorphism.

(i) Show that if $M \in \text{Sub } L$, then $f(M) \in \text{Sub } K$.

(ii) Show that if $N \in \text{Sub } K$, then $f^{-1}(N) \in \text{Sub}_0 \ L$, where $\text{Sub}_0 L = \text{Sub } L \cup \emptyset$.

(b) Let L be a lattice.

(i) Assume that $b \leq a \leq b \vee c$ for $a, b, c \in L$. Show that $a \vee c = b \vee c$.

(ii) Show that the operations $\vee$ and $\wedge$ are isotone in L, i.e. $b \leq c \Rightarrow a \wedge b \leq a \wedge c$ and $a \vee b \leq a \vee c$.

(c) Let L and M be lattices. Show that the product $L \times M$ is a lattice under the operations $\vee$ and $\wedge$ defined as

$$(x_1,y_1) \vee (x_2,y_2) := (x_1 \vee x_2,\ y_1 \vee y_2),$$

$$(x_1,y_1) \wedge (x_2,y_2) := (x_1 \wedge x_2,\ y_1 \wedge y_2)$$

4. (a) Let L be a distributive lattice. Show that $\forall x, y, z \in L$, the following laws are equivalent :

(i) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

(ii) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

(b) Define modular lattices. Show that every distributive lattice is modular. Is the converse true? Give arguments in support of your answer.

(c) (i) Prove that for any two elements x, y in a lattice L, the interval

$[x,y] := \{a \in L | x \leq a \leq y\}$ is a sublattice of L.

(ii) Let f be a monomorphism from a lattice L into a lattice M. Show that L is isomorphic to a sublattice of M.

5. (a) (i) Prove that $(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$ for all x, y in a Boolean algebra.

Deduce that $x \leq y \Leftrightarrow x' \geq y'$ for all $x, y \in B$.

(ii) Show that the lattice $B = (\{1,2,3,6,9,18\}, \text{gcd}, \text{lcm})$ of all positive divisors of 18 does not form a Boolean algebra.

(b) Find the conjunctive normal form of

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'$$

(c) Use a Karnaugh Diagram to simplify

$$p = x_1x_2x_3 + x_2x_3x_4 + x_1'x_2x_4' + x_1'x_2x_3x_4' + x_1'x_2x_4'$$

6. (a) Use the Quine-McCluskey method to find the minimal form of

$$wxyz' + wxy'z' + wx'yz + wx'yz' + w'x'yz + w'x'yz' + w'x'y'z$$

(b) Draw the contact diagram and determine the symbolic representation of the circuit given by

$$p = x_1x_2(x_3+x_4) + x_1x_3(x_5+x_6)$$

(c) Give mathematical models for the following random experiments

(i) when in tossing a die, all outcomes and all combinations are of interest.

(ii) when tossing a die, we are only interested whether the points are less than 3 or greater than or equal to 3.

4 2

[This question paper contains 4 printed pages.] 20m

Your Roll No..............

Sr. No. of Question Paper : 4350 G

Unique Paper Code : 32351301

Name of the Paper : Theory of Real Functions

Name of the Course : B.Sc. (H) Mathematics (LOCF)

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.
3. All questions are compulsory.

1. (a) Let $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A and $f: A \to \mathbb{R}$, then define limit of function f at c. Use $\epsilon - \delta$ definition to show that $\lim_{x \to 2} \frac{1}{1-x} = -1$. (6)

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$.

P.T.O.

Show that f has a limit at x = 0. Use sequential criterion to show that f does not have a limit at c if c ≠ 0. (6)

(c) Show that $\lim_{x \to 0} \cos\left(\frac{1}{x^2}\right)$ does not exist in $\mathbb{R}$ but

$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$. (6)

2. (a) Let $A \subseteq \mathbb{R}$, let f: $A \to \mathbb{R}$ and g: $A \to \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Show that if

$\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ then $\lim_{x \to c} (fg)(x) = LM$.

(6)

(b) Evaluate the limit $\lim_{x \to 1+} \frac{x}{x-1}$. (6)

(c) Let $A \subseteq \mathbb{R}$, f: $A \to \mathbb{R}$ and $c \in \mathbb{R}$. Show that the following conditions are equivalent-

(i) f is continuous at c.

(ii) For every sequence $\langle x_n \rangle$ in A that converges to c, the sequence $\langle f(x_n) \rangle$ converges to f(c). (6)

3. (a) Let A, $B \subseteq \mathbb{R}$ and let f: $A \to \mathbb{R}$ and g = $B \to \mathbb{R}$ be functions such that $f(A) \subseteq B$. if f is continuous at a point $c \in A$ and g is continuous at $b \in B$,

then show that the composition function $g \circ f: A \to \mathbb{R}$ is continuous at c. Also, show that the function $f(x) = \cos(1 + x^2)$ is continuous on $\mathbb{R}$. (7½)

(b) State and prove Maximum-Minimum Theorem for continuous functions on a closed and bounded interval. (7½)

(c) State Bolzano's Intermediate value theorem. Show that every polynomial of odd degree with real coefficients has at least one real root. (7½)

4. (a) Let $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$. Show that if f is continuous at $c \in A$ then $|f|$ is continuous at c. Is the converse true? Justify your answer. (6)

(b) Let $I = [a, b]$ and $f: I \to \mathbb{R}$. Show that if f is continuous on I then it is uniformly continuous on I. (6)

(c) Show that $f(x) = \sin x$ is uniformly continuous on $\mathbb{R}$ and the function $g(x) = \sin\left(\frac{1}{x}\right)$, $x \neq 0$ is not uniformly continuous on $(0, \infty)$. (6)

5. (a) Let $I \subseteq \mathbb{R}$ be an interval, let $c \in I$, and let $f: I \to \mathbb{R}$ and $g: I \to \mathbb{R}$ be functions that are differentiable at c. Prove that the function fg is differentiable at c, and $(fg)' = f'(c)g(c) + f(c)g'(c)$. (6)

(b) Let $g: \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x)=\begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Show that g is differentiable for all $x \in \mathbb{R}$.

Also, show that the derivative g' is not continuous at $x = 0$. (6)

(c) Suppose that f is continuous on a closed interval $I = [a, b]$, and that f has a derivative in the open interval (a, b). Prove that there exists at least one point c in (a, b) such that $f(b) - f(a) = f'(c)(b - a)$.

Suppose that $f: [0, 2] \to \mathbb{R}$ is continuous on $[0,2]$ and differentiable on $(0,2)$ and that $f(0) = 0$, $f(2) = 1$. Show that there exists $c_1 \in (0,2)$ such $f'(c_1) = 1/2$. (6)

6. (a) Find the points of relative extrema of the function $f(x) = 1 - (x - 1)^{2/3}$, for $0 \leq x \leq 2$. (6)

(b) Let I be an open interval and let $f: I \to \mathbb{R}$ has a second derivative on I. Then show that f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. (6)

(c) Obtain Taylor's series expansion for the function $f(x) = \sin x$, $\forall x \leq \mathbb{R}$. (6)

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27/12

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4332 **G**

Unique Paper Code : 32351501

Name of the Paper : BMATH511 – Metric Spaces

Name of the Course : **B.Sc. (Hons) Mathematics (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Let (X, d) be a metric space. Show that (X, d^*) is a metric space where

$d^*(x, y) = \min\{1, d(x,y)\}, \forall x, y \in X.$ (6)

(b) (i) Let (X, d) be a metric space. Let $\langle x_n \rangle$ and $\langle y_n \rangle$ be sequences in X such that $\langle x_n \rangle$ converges to x and $\langle y_n \rangle$ converges to y. Prove that $d(x_n, y_n)$ converges to d(x, y). (2)

P.T.O.

(ii) Prove that if a Cauchy sequence of points in a metric space (X, d) contains a convergent subsequence, then the sequence converges to the same limit as the subsequence. (4)

3.

(c) (i) Let X = ℕ, the set of natural numbers. Define $d(m, n) = \left|\frac{1}{m} - \frac{1}{n}\right|$; m, n ∈ X. Show that (X, d) is an incomplete metric space. (4)

(ii) Is the metric space (X, d) of the set X of rational numbers with usual metric d a complete metric space? Justify. (2)

2. (a) (i) Define an open set in a metric space (X, d). Show that every open ball in (X, d) is an open set. Is the converse true? Justify. (4)

(ii) Let S(x, r) be an open ball in a metric space (X, d). Let A be a subset of X such that diameter of A, d(A) < r and S(x, r) ∩ A ≠ Ø. Show that A ⊆ S(x, 2r). (2)

4.

(b) Let (X, d) be a metric space and A_1 and A_2 be subsets of X. Prove that $\overline{(A_1 \cup A_2)} = \overline{A_1} \cup \overline{A_2}$. Is the closure of the union of an arbitrary family of the subsets of X equal to the union of the closures of the members of the family? Justify. (6)

(c) Prove that a subspace of a complete metric space is complete if and only if it is closed. (6)

3. (a) Let (X, d_X) and (Y, d_Y) be two metric spaces. Show that a mapping $f: X \to Y$ is continuous if and only if for every subset F of Y, $(f^{-1}(F))^{\circ} \supseteq f^{-1}(F^{\circ})$. (6)

(b) (i) Let (X, d) be a metric space and A be a non-empty subset of X. Let $f(x) = d(x, A) = \inf \{d(x, a), a \in A\}$, $x \in X$. Show that f is uniformly continuous over X. (4)

(ii) Is a continuous function over a metric space always uniformly continuous? Justify. (2)

(c) Let (X, d) be a metric space and $f: X \to \mathbb{R}^n$ be a function defined by $f(x) = (f_1(x), f_2(x) \ldots f_n(x))$, where $f_k: X \to \mathbb{R}$, $1 \le k \le n$ is a function. Show that f is continuous on X if and only if for each k, f_k is continuous on X. (6)

4. (a) Define homeomorphism between two metric spaces. Show that the image of a complete metric space under homeomorphism need not be complete. (6.5)

(b) Let d_1 and d_2 be two metrics on a non-empty set X. Show that d_1 and d_2 are equivalent if and only if the identity mapping $I: (X, d_1) \to (X, d_2)$ is a homeomorphism. (6.5)

(c) Let $T: X \rightarrow X$ be a contraction of a complete metric space (X, d). Show that T has a unique fixed point. (6.5)

5. (a) Show that the subset $A \subseteq \mathbb{R}^2$, where (6.5)

$A = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 \geq 9\}$ is disconnected.

(b) Let $I = [-1, 1]$ and let $f: I \rightarrow I$ be continuous, then show that there exists a point $c \in I$ such that $f(c) = c$. Discuss the result if $I = [-1,1)$. (4+2.5)

(c) Let (X, d_X) be a connected metric space and f be a continuous mapping from (X, d_X) onto (Y, d_Y). Prove that (Y, d_Y) is also connected. Does there exist an onto continuous map $g: [0,1] \rightarrow [2, 3] \cup [4, 5]$? Justify your answer. (6.5)

6. (a) Let f be a continuous function from a compact metric space (X, d_X) to a metric space (Y, d_Y), then prove that f is uniformly continuous on X. (6.5)

(b) Let (X, d) be a metric space and Y be a compact subset of (X, d). Then prove that Y is closed and bounded. Give an example of a closed and bounded subset of a metric space which fails to be compact. (4+2.5)

(c) State finite intersection property. Show by using the finite intersection property that $(\mathbb{R}, d)$ with usual metric is not compact. (2+4.5)

[This question paper contains 4 printed pages.]

18 JUN 2024

Your Roll No..............

Sr. No. of Question Paper : 4518 **G**

Unique Paper Code : 32351303

Name of the Paper : Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **Five** questions from each section.
4. **All** questions carry equal marks.

Section I

1. Find the following limits :

(i) $\lim_{(x,y)\to(0,0)} \left(1+x^2+y^2\right)^{\frac{1}{x^2+y^2}}$

(ii) $\lim_{(x,y)\to(0,0)} x \log\sqrt{\left(x^2+y^2\right)}$

2. Find an equation for each horizontal tangent-plane to the surface

$$z = 5 - x^2 - y^2 + 4y$$

3. The output at a certain factory is $Q = 150K^{\frac{2}{3}}L^{\frac{1}{3}}$ where K is the capital investment in units of \$1000, and L is the size of Labor force measured in worker-hours. The current capital investment is \$500,000 and 150 worker hours of Labor are used. Estimate the change in output that results when capital investment is increased by \$500 and Labor is decreased by 4 worker-hours.

4. Let $w = f(t)$ be a differentiable function of t where $t = (x^2 + y^2 + z^2)^{1/2}$. Show that

$$(dw/dt)^2 = (\partial w/\partial x)^2 + (\partial w/\partial y)^2 + (\partial w/\partial z)^2.$$

5. Let $f(x, y, z) = xyz$ and let $\hat{u}$ be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1, -1, 2)$ in the direction of $\hat{u}$.

6. Find the absolute extrema of the function $f(x, y) = e^{x^2 - y^2}$ over the disk $x^2 + y^2 \le 1$.

Section II

1. Evaluate the double integral $\iint_D \frac{dA}{y^2+1}$ where D is triangle bounded by x=2y, y=–x and y=2.

2. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{(x^2+y^2)}\, dy\, dx$ by converting to polar coordinates.

3. Find the volume of tetrahedron T bounded by plane $2x + y + 3z = 6$ and co-ordinate planes.

4. Use spherical co-ordinates to verify that volume of a half sphere of radius R is $\frac{2}{3}\pi R^3$.

5. Use cylindrical co-ordinates to compute the integral $\iiint_D z\left(x^2+y^2\right)^{-\frac{1}{2}} dx\, dy\, dz$ where D is the solid bounded above by the plane z=2 and below by the surface $2z = x^2 + y^2$.

6. Use a suitable change of variables to compute the double integral $\iint_D \left(\frac{x-y}{x+y}\right)^2 dy\, dx$, where D is the triangular region bounded by line $x + y = 1$ and co-ordinate axes.

P.T.O.

Section III

1. Find the mass of a wire in the shape of curve C: $x = 3 \sin t$, $y = 3 \cos t$, $z = 2t$ for $0 \le t \le \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = z$.

2. Find the work done by force $\vec{F} = x\hat{i} + y\hat{j} + (xz - y)\hat{k}$ on an object moving along the curve C given by $R(t) = t^2\hat{i} + 2t\hat{j} + 4t^3\hat{k}$.

3. Use Green's theorem to find the work done by the force field $\vec{F}(x,y) = y^2\hat{i} + x^2\hat{j}$ when an object moves once counterclockwise around the circular path $x^2 + y^2 = 2$.

4. State and prove Green's Theorem.

5. Evaluate $\oint (2xy^2z\,dx + 2x^2yz\,dy + (x^2y^2 - 2z)\,dz)$ where C is the curve given by $x = \cos t$, $y = \sin t$, $z = \sin t$, $0 \le t \le 2\pi$ traversed in the direction of increasing t.

6. Use divergence theorem to evaluate $\iint_S \vec{F}.N\,ds$ where

$$\vec{F} = (x^5 + 10xy^2z^2)\hat{i} + (y^5 + 10yx^2z^2)\hat{j} + (z^5 + 10zy^2x^2)\hat{k}$$

and S is closed hemisphere surface $z = \sqrt{1 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 1$ in x-y plane.

[This question paper contains 4 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 4386 G

Unique Paper Code : 32351502

Name of the Paper : Group Theory – II

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Question No. **1** has been divided into **10** parts and each part is of **1.5** marks.
4. Each question from Q. Nos. **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each question.

1. State true (T) or false (F). Justify your answer in brief.

   (i) Let G be a finite group of order 147 then it has a subgroup of order 49.

   (ii) There is a simple group of order 102.

P.T.O.

(iii) Dihedral Group D_{12} (having 24 elements) is isomorphic to the symmetric group S_5.

(iv) The action $z \cdot a = z + a$ of the additive group of integers Z on itself is faithful.

(v) The external direct product $G \oplus H$ is cyclic if and only if groups G and H are cyclic.

(vi) Trivial action is always faithful.

(vii) The group of order 27 is abelian.

(viii) The external direct product $Z_2 \oplus Z_6$ is cyclic.

(ix) Every Sylow p-subgroup of a finite group has order some power of p.

(x) A p-group is a group with property that it has atleast one element of order p.

2. (a) Prove that for every positive integer n, Aut $(Z_n) \cong U(n)$.

(b) Define Automorphism Aut(G) of a group G and Inner Automorphism Inn(G) of the group G induced by an element 'a' of G. Prove that Aut(Z_5) is isomorphic to U(5), where U(5) = {1, 2, 3, 4} is group under the multiplication modulo 5.

(c) Define characteristic subgroup of G. Prove that every subgroup of a cyclic group is characteristic.

3. (a) Prove that the order of an element of a direct product of finite number of finite groups is the least common multiple of the orders of the components in the elements. Find the largest possible order of an element in $Z_{30} \oplus Z_{20}$.

(b) Prove that if a group G is the internal direct product of finite number of subgroups H_1, H_2,.... H_n then G is isomorphic to $H_1 \oplus H_2 \oplus H_3 \ldots \oplus H_n$.

(c) Let G is an abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G.

4. (a) Show that the additive group R acts on x,y plane R×R by r.(x,y)=(x+ry,y).

(b) Let G be a group acting on a non-empty set A. Define

(i) kernel of group action

(ii) Stabilizer of a in G, for a ∈ A

(iii) Prove that kernel is a normal subgroup of G.

(c) Define the permutation representation associated with action of a group on a set. Prove that the kernel of an action of group G on a set A is the same as the kernel of the corresponding permutation representation of the action.

5. (a) Let G be a group acting on a non-empty set A. If $a, b \in A$ and $b = g \cdot a$ for some $g \in G$. Prove that $G_b = g\, G_a\, g^{-1}$ where G_a is stabilizer of a in G. Deduce that if G acts transitively on A then kernel of action is $\cap_{g \in G}\, g\, G_a\, g^{-1}$.

(b) Define the action of a group G on itself by conjugation. Prove it is a group action. Also find the kernel of this action.

(c) If G is a group of order pq, where p and q are primes, $p < q$, and p does not divide q-1, then prove that G is cyclic.

6. (a) State the Class Equation for a finite group G. Find all the conjugacy classes for quaternion group Q_8 and also, compute their sizes. Hence or otherwise, verify the class equation for Q_8.

(b) Use Sylow theorems to determine if a group of order 105 is not simple.

(c) State and prove Embedding theorem and use it to prove that a group of order 112 is not simple.

[This question paper contains 8 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 4469 G

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All **six** questions are compulsory.

3. Attempt any **two** parts from each question.

4. Use of non-programmable scientific calculator is allowed.

1. (a) Discuss the order of convergence of the Newton Raphson method. (6)

(b) Perform three iterations of the Bisection method in the interval (1, 2) to obtain root of the equation $x^3 - x - 1 = 0$. (6)

(c) Perform three iterations of the Secant method to obtain a root of the equation $x^2 - 7 = 0$ with initial approximations $x_0 = 2$, $x_1 = 3$. (6)

2. (a) Perform three iterations of False Position method to find the root of the equation $x^3 - 2 = 0$ in the interval (1, 2). (6.5)

(b) Find a root of the equation $x^3 - 5x + 1 = 0$ correct up to three places of decimal by the Newton's

Raphson method with $x_0 = 0$. In how many iterations does the solution converge? Also write down the order of convergence of the method used. (6.5)

(c) Explain the secant method to approximate a zero of a function and construct an algorithm to implement this method. (6.5)

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \quad 4 \quad 1]^T$. (6.5)

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$5x_1 + x_2 + 2x_3 = 10$$

$$-3x_1 + 9x_2 + 4x_3 = -14$$

$$x_1 + 2x_2 - 7x_3 = -33$$

Take the initial approximation as $X^{(0)} = (0,0,0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. (6.5)

4. (a) Construct the Lagrange form of the interpolating polynomial from the following data :

x	0	1	3
f(x)	1	3	55

(6)

(b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial.

x	0	1	2	3
y	–1	0	15	80

Hence, estimate the value of f(1.5). (6)

(c) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data :

P.T.O.

x	−1	0	1	2
f(x)	3	−1	−3	1

(6)

5. (a) Derive second-order backward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}.$$ (6)

(b) Use the formula

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

to approximate the second derivative of the function $f(x) = e^x$ at $x_0 = 0$, taking h = 1, 0.1, 0.01 and 0.001. What is the order of approximation.

(6)

(c) Approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 0$ using the first order forward difference formula taking $h = ½, ¼$ and $1/8$ and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the trapezoidal rule, approximate the value of the integral $\int_3^7 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Derive the Simpson's $1/3^{rd}$ rule to approximate the integral of a function. (6.5)

(c) Apply the modified Euler method to approximate the solution of the initial value problem

P.T.O.

$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 2, x(1) = 1$ taking the step size as

h = 0.5. (6.5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4470

Unique Paper Code : 32357502

Name of the Paper : DSE-1 Mathematical Modelling and Graph Theory

Name of the Course : **B.Sc. (H) Mathematics – (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) (i) Determine whether x = 0 is an ordinary point, a regular singular point or an irregular singular point of the differential equation

$$x^2y'' + (6 \sin x)y' + 6y = 0.$$

(ii) Find the Laplace transform of the function $f(t) = \sin 3t \cos 3t$

P.T.O.

(iii) Find the inverse Laplace transform of the function $F(s) = \frac{9+s}{4-s^2}$. (6)

(b) Use Laplace transforms to solve the initial value problem :

$x'' + 6x' + 25x = 0$; $x(0) = 2$, $x'(0) = 3$ (6)

(c) Find two linearly independent Frobertius series solutions of (6)

$$4xy'' + 2y' + y = 0$$

(d) Find general solutions in powers of x of the differential equation. State the recurrence relation and the guaranteed radius of convergence.

$$5y'' - 2xy' + 10y = 0$$ (6)

2. (a) Explain Linear Congruence method for generating random numbers. Does this method have any drawback? Explain with the help of an example. (6)

(b) Use of Monte Carlo simulation to approximate the area under the curve y = cos x over the interval $-\pi/2 \le x \le \pi/2$, where $0 \le \cos x \le 2$. (6)

(c) Using algebraic analysis, solve the following:

Maximize : $x + 2y$

subject to $5x + 2y \le 10$

$2x + 3y \le 6$

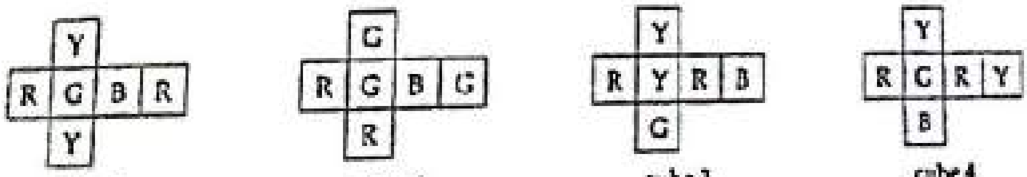
$x_1, x_2 \ge 0$. (6)

(d) Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	20	30	15	120	25
Unload time	55	45	60	75	80

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue and total time in which docking facilities are idle. (6)

3. (a) Find the solution to the four-cubes problem for the following set of cubes. (6)



cube 1 cube 2 cube 3 cube 4

(b) Define semi-Eulerian trail. Prove that a bipartite graph with an odd number of vertices is not Hamiltonian. (6)

(c) Prove that there is no Knight's tour on a 7x7 Chessboard. (6)

P.T.O.

(d) State Handshaking lemma. Use it to prove that in any graph, the number of vertices of odd degree is even. (6)

4. (a) Use the factorization :

$s^4 + 4a^4 = (s^2 - 2as + 2a^2)\ (s^2 + 2as + 2a^2)$

and apply inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s}{s^4 + 4a^4}\right\} = \frac{1}{2a^2}\sinh\ at\ \sin\ at \qquad (7)$$

(b) Using Simplex method, solve the following linear programming problem :

Maximize $z = 5x_1 + 3x_2$

subject to $-3x_1 - 5x_2 \geq -15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$ (7)

(c) (i) State Ore's Theorem. (2)

(ii) Show that $L\ \{t\ \mathrm{Cos}(kt)\} = \dfrac{s^2 - k^2}{\left(s^2 + k^2\right)^2}$. (5)

(d) Define Cube graphs. Write the number of vertices and number of edges in a cube graph Q_k. Draw Q_1, Q_2 and Q_3. (7)

[This question paper contains 8 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 4586 G

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the given **eight** questions are compulsory to attempt.

3. Do any **two** parts from each of the given eight questions.

4. Marks for each part are indicated on the right in brackets.

SECTION 1

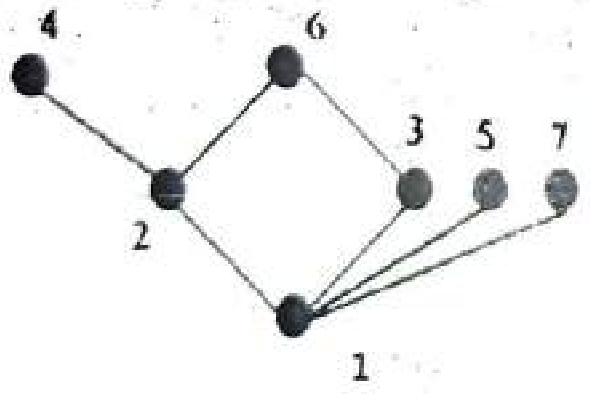
1. (a) Let P = {a, b, c, d, e, f, u, v}. Draw the Hasse diagram for the partially ordered set (P; ≤), where the relations are given by:

$v < a,\ v < b,\ v < c,\ v < d,\ v < e,\ v < f,\ v < u,$
$a < c,\ a < d,\ a < e,\ a < f,\ a < u,$
$b < c,\ b < d,\ b < e,\ b < f,\ b < u,$
$c < d,\ c < e,\ c < f,\ c < u,$
$d < e,\ d < f,\ d < u,$
$e < u,\ f < u$

(2½)

(b) Give an example of a partially ordered set (P; ≤) which is neither a chain nor an anti chain. Justify with suitable arguments as to why this partially ordered set (P; ≤) is not a chain and not an antichain. (2½)

(c) Consider the diagram below of the ordered subset P = {1, 2, 3, 4, 5, 6, 7} of (N_0; ≤), where (N_0; ≤) is the ordered set of non-negative integers ordered by relation ≤ on N_0 as: For m, n ∈ N_0, m ≤ n if m divides n, that is, if there exists k ∈ N_0: n=km.



For the following subsets of P, find the following meet/join as indicated. Either specify the meet/join if it exists or indicate why it fails to exist.

(i) meet and join of subset {2,3,5}

(ii) meet and join of subset {2,3,6}

(iii) join of P (2½)

2. (a) Show that an order isomorphism for two ordered sets P and Q is a bijection, but the converse is not true. (3)

(b) Let P and Q be ordered sets. Prove that: $(a_1,b_1) \prec (a_2,b_2)$ in $P \times Q$ iff ($a_1 = a_2$ and $b_1 \prec b_2$) or ($a_1 \prec a_2$ and $b_1 = b_2$) (3)

(c) Let P, Q and R be ordered sets and let $\varphi: P \to Q$ and $\psi: Q \to R$ be order preserving maps. Then show that the composite map: $\psi o \varphi: P \to R$ given by:
$(\psi o \varphi)(x) = \psi((\varphi(x))$ for $x \in P$, is also an order preserving map. (3)

SECTION II

3. (a) Let $(L, \leq)$ be a lattice with respect to the order relation $\leq$. For the operations $\wedge$ and $\vee$ defined on L as:

$x \wedge y = \inf(x, y)$, $x \vee y = \sup(x, y)$

show that $(L, \wedge, \vee)$ is an algebraic lattice, that is the associative laws, commutative laws, idempotency laws and absorption laws hold.

(5)

(b) Define a lattice. Let $D_{24} = \{1,2,3,4,6,8,12,24\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '$\leq$' defined on D_{24} by $m \leq n$ iff m divides n, then show that D_{24} forms a lattice. (5)

(c) Let L_1 and L_2 be modular lattices. Prove that the product $L_1 \times L_2$ is a modular lattice. (5)

4\. (a) Let L and K be lattices and f: L → K be a homomorphism. Then show that the following are equivalent

(i) f is order-preserving

(ii) $(\forall\ a, b \in L)$, $f(a \vee b) \geq f(a) \vee f(b)$

(5½)

(b) Let L be a lattice and let a, b, c Î L. Then show that :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ (5½)

(c) Define distributive lattice. Prove that homomorphic image of distributive lattice is distributive. (5½)

SECTION III

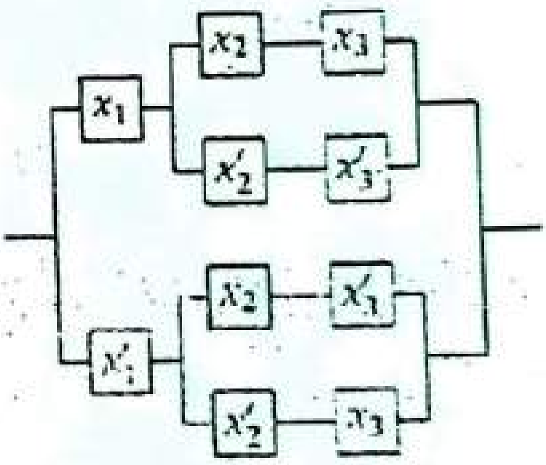
5. (a) Find the disjunctive normal form for (5½)

$(x_1 + x_2 + x_3)(x_1x_2 + x'_1x_3)'$

(b) Using Karnaugh diagram, simplify the expression

$x_3(x_2 + x_4) + x_2x'_4 + x'_2x'_3x_4$ (5½)

(c) Find symbolic gate representation for (5½)



6. (a) Find the conjunctive normal form of $x_1(x_2+x_3)' + (x/x'_2\ x'_3)x_1$ in three variables. (5)

(b) If B is the set of all positive divisors of 110, then show that (B, gcd, lcm) is a Boolean Algebra. (5)

(c) Find minimal form of the polynomial:

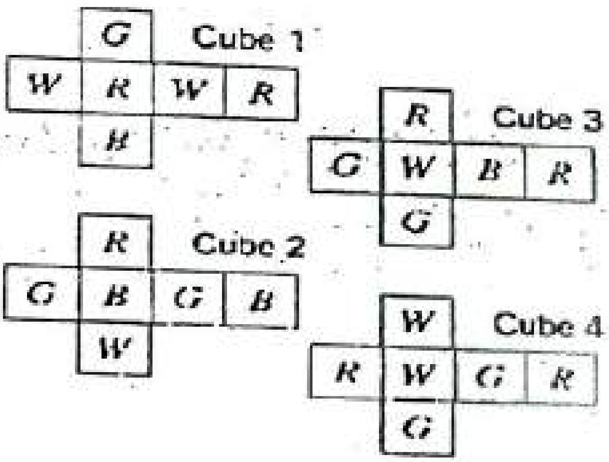
$$f = x'y + x'y'z + xy'z' + xy'z$$

using Quine's McCluskey method. (5)

SECTION IV

7. (a) What is the Three houses- Three Utilities Problem? How can it be formulated using graphs? Does this problem have a solution? (5½)

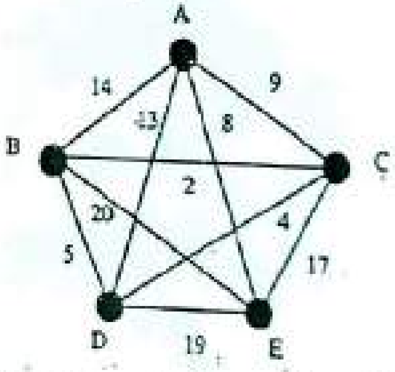
(b) Given four cubes as shown below (the cubes are cut along a few edges, then opened up and flattened) :



Find the solution, if it exists, for the game of "Instant Insanity" using the above four cubes, where the 6 faces of each of the four cubes have been coloured using four colours red(R), green(G), blue(B) and white(W). (5½)

(c) Explain the Konigsberg bridge problem and formulate it using a corresponding graph. Does the problem have a solution? Give reasons for your answer. (5½)

8. (a) Apply Improved Version of Dijkstra's Algorithm to find shortest distances from vertex A to all other vertices.



(5½)

(b) Consider the two graphs: the pentagon and the star as given below. Compute their adjacency matrices. Are they isomorphic to each other? If yes, exhibit an isomorphism between them. If not, then give suitable argument. (5½)

[This question paper contains 4 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 4587

Unique Paper Code : 32357506

Name of the Paper : DSE-II Cryptography and Network Security

Name of the Course : B.Sc. (H) Mathematics

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **five** parts from questions **1**, each part carries **3** marks.

4. Attempt any **two** parts from questions **2** to **6**, each part carries **6** marks.

P.T.O.

1. (a) Write a short note on Active attack in computer security.

   (b) What is a transposition cipher? Write the message

   'a t t a c k p o s t p o n e d u n t i l t w o a m'

   row by row and encrypt it with the key '4 3 1 2 5 6 7'.

   (c) What is Euler's Totient function? Find φ(143) and φ(71).

   (d) Briefly describe MixColumns transformation of AES.

   (e) Is (5, 12) a point on the elliptic curve $y^2 = x^3 + 4x - 1$ over real numbers.

   (f) Define direct digital signature.

2. (a) Explain Symmetric Cipher model with the help of a diagram.

   (b) Encrypt the message 'CRYPTOGRAPHY' using Playfair Cipher with the key 'algorithm'. Write the rules while encrypting the message.

   (c) Explain the Feistel Decryption process with the help of a diagram.

3. (a) State the Chinese Remainder theorem and hence solve the following system of linear congruence relations :

$x \equiv 2 \pmod 5$, $x \equiv 5 \pmod 8$ and $x \equiv 4 \pmod{37}$.

(b) State the Fermat's theorem. Find the remainder when $(300)^{3000}$ is divisible by 1001.

(c) Explain the Data Encryption Standard (DES) with the help of a diagram.

4. (a) Find the multiplicative inverse of 550 mod (1759) where 1759 is a prime number.

(b) Identify $GF(2^8)$ with the field of polynomial over $GF(2)$ modulo $m(x) = x^8 + x^4 + x^3 + x + 1$. If the byte $b_7\, b_6\, b_5\, b_4\, b_3\, b_2\, b_1\, b_0$ represent the polynomial $b_7 x^7 + b_6 x^6 + b_5 x^5 + b_4 x^4 + b_3 x^3 + b_2 x^2 + b_1 x + b_0$ in the field, find the product $f(x)\, g(x)$ where $f(x) = (01010011)$ and $g(x) = (10111010)$ are elements of the field.

(c) Perform encryption and decryption using the RSA algorithm for $p = 3$, $q = 11$, $e = 7$ and $M = 14$.

5. (a) On the elliptic curve over Z_{23}, $y^2 = x^3 + x + 1$, Let $P = (13,7)$ and $Q = (9,7)$. Find $P + Q$ and $2P$.

(b) Describe a hash function. Give three applications of the hash function, clearly specifying the role of the hash function in each application.

(c) Describe E-mail Protocols.

6. (a) Perform the Elgamal signature scheme with $q = 19$, $\alpha = 10$, $x_A = 16$, $k = 5$ and $m = 14$.

(b) Write a short note on RFC 5322 and Multipurpose Internet Mail Extensions (MIME).

(c) Write all wireless environment components. Explain wireless Network Threats.